THE USE OF PENDULUMS IN OSCILLATIONS CONTROL OF “ROCKING-BLOCK” TYPE BUILDINGS

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ABSTRACT

This research deals with the dynamical evolutions played by a simple mechanical model. The model is constituted of a parallelepiped standing on a horizontal plane. As the plane performs vibrational displacements the parallelepiped begins to evolve its positions and, this “parallelepiped movements” constitutes the subject of our concern. Plane movement is assumed so, notwithstanding we name it as a parallelepiped, or “rocking block”, the structure is actually schematized as a “physical rectangle”. From this simple model and its elaboration by adding a planar physical pendulum and some damping between the parallelepiped and the ground surface, some interesting results have been achieved.

Keywords: rocking block; vibrations control; earthquakes.

1 INTRODUCTION

The technical scenarios that provoke these studies originate from earthquakes. Interactions between buildings and earthquakes have been studied for a long time and there is much literature about them. In Figure 1, examples of collapses due to earthquakes are reported. These collapses evidence a rocking-block behavior of the structures, namely rigid movement of the block with respect to the ground, as illustrated in the scheme of Figure 2. When vibrations of buildings may be assumed (and proved) to be elastic and linear, one may think to reduce them by applying some “tuned mass”, Clough and Penzien (1993). It is in fact possible to absorb energy at certain frequencies (or ranges) by coupling an oscillator with the main structure. This oscillator is often coupled by a spring-damper system, in order to sink energy from the system. However, when considering masonry buildings or no-tension structures in general, the situation changes considerably. Consider for example a slender tower, such as the masonry chimney illustrated in Figure 3. When the tower oscillates, e.g., under the forcing of an earthquake, a plausible model for its motion is that of a rigid parallelepiped tilting back and forth on a horizontal plane. In this situation oscillations are no longer linear.

Following the seminal work by Housner in 1963, the problem of the motion of an oscillating parallelepiped-shaped rigid body, the so-called “rocking-block” model, has been widely investigated. The main issue investigated in this and other earlier studies was the performance of a rigid body against the base motion dynamics, e.g., an earthquake, assuming the block to be a building model, Priestly et al. (1978), Aslam et al. (1980), Yim et al. (1980), Ishiyama (1980), Spanos and Koh (1984), Lipscombe and Pellegrino (1993), Hogan (1994, 1994b).
The work by Housner was devoted to the explanation of the collapse of many bulky structures during the catastrophic Chilean earthquake of May 1960. From the solution of the free oscillation problem of a rigid rocking block bouncing on its vertices, the overturning by a constant, horizontal acceleration by a single sine pulse, and by an earthquake-type excitation, evidenced an unexpected scale effect that makes the larger of two geometrically similar blocks more stable than the smaller block. Consequently, it was demonstrated that tall slender structures show a better than expected stability.

In subsequent years, other works developed the study of the dynamics of this simple building model by introducing some other quantities and/or situations to improve the physical understanding of the model itself. The works by Hogan (1989, 1990, 1994a, 1994b) refined the pioneering study by Housner, assuming a similar rocking block model, and analyzing in depth its dynamics under harmonic forcing. In these works, the simple block model is shown to possess extremely complicated dynamics, including chaos. The existence and the form of subharmonic and asymmetric responses are explained and validated by experimental works by Wong and Tso (1989).

A novel formulation for the rocking motion of a rigid block on a rigid foundation is presented in Prieto and Lourenço (2005). The traditional piecewise equations are replaced by a single ordinary differential equation, and damping effects are no longer introduced by means of a coefficient of restitution, but are understood as the presence of impulsive forces. The results are in agreement with the classical formalism, and can be set in direct analogy with either a two-body central problem in the complex plane, or an inverted pendulum through simple variable transformations.

The rigid block sliding and bouncing effects are studied by Andreaus and Casini (1999). In this work, the “contact-impact” problem of a rigid block colliding on a frictional base is analyzed. Refined analytical stress-strain relations in either normal or tangential directions with respect to the contact surfaces, allow the uplifting and hysteretic damping in the normal direction, and the coupling between shear strength and compression force, friction dissipation and accumulating damage in the tangential direction to be accounted for. Numerical simulation revealed the dependence of energy reduction upon block slenderness and friction angle; due to the bouncing phenomenon, great horizontal displacements with a high value of the friction angle can be revealed, and the existence of a friction critical value below which the block remains at rest is demonstrated.

Figure 1: Collapses of the “rocking-block”-type due to earthquakes.
A rocking-block model has also been employed to study the response of structures, such as columns or monumental walls subjected to frictionless multiple impacts, Yilmaz et al. (2009). The embraced methodology is based on the use of impulse-momentum methods in conjunction with the energetic coefficient of restitution, and yields energetically consistent solutions. It has been shown that one may have single and/or simultaneous multiple collisions at the surface contact points. Furthermore, the block is assumed to reproduce and facilitate the study of the behavior of monumental structures such as ancient columns, see Sinopoli (1991). The aim of this work is the identification of an analytical model by which to study the dynamic behavior of either a monolithic or multi-block stone column, excited by a sine wave ground motion. The work demonstrates an operative limit of the model in relation to the problem that the system exhibits some kind of unpredictability. In any case, criteria for a stability analysis of the motions are defined.

Recently, the dynamic response of the rocking block subjected to base excitation has been revisited to offer new closed-form solutions and original similarity laws that shed light on the fundamental aspects of the original model by Dimitrakopoulos and DeJong (2012). The focus is on the transient dynamics of the rocking block under finite-duration excitations. In the process, limitations of standard dimensional analysis related to the orientations of the involved physical quantities are revealed. The work reveals that the nonlinear and non-smooth rocking response to pulse-type ground motion can be directly determined, and need only be scaled by the intensity and frequency of the excitation.

Finally, similar to the concern of the present work, the problem of vibration control is investigated by Housner et al. (1997) and by Lenci and Rega (2006). In these works, a systematic theoretical investigation of control/anti-control of the nonlinear dynamics of a rocking block has been made through the analysis of two curves: the heteroclinic bifurcation and the immediate overturning thresholds, characterizing the system response in excitation parameters space in terms of overturning behavior. The differences between control and anti-control are shown to be mostly of a technical nature, and consist of the different definition of the amplitude of the excitation and in the different nature of the optimization problems. These have been solved in closed form by determining the optimal excitations with an increasing number of superharmonics.

The preliminary questions addressed by this work are: i) Is it possible to give some parameterization of these oscillations? ii) Is it possible to tune a pendulum with this tower so that oscillations are reduced and controlled? iii) Is it possible to give some sort of frequency response (with all the limitations imposed by non-linearity)?
The simple model studied in the following sections partially answers these questions. The model is somewhat simplified. Plane movement is assumed and therefore, we name it as a parallelepiped, or “rocking block”; the structure is actually schematized as a “physical rectangle”. From this simple model and its elaboration through adding a planar physical pendulum and some damping between the parallelepiped and the ground surface, some interesting results have been achieved.

2 THEORETICAL APPROACH: LAGRANGIAN OF THE SYSTEM

2.1 Statement of the problem

With the nomenclature reported in Figure 3 we can write the following set of equations dealing with the potential and kinetic energies of a block $M$ rocking on its corners, to which a pendulum of mass $m$ is added.

\[
\begin{align*}
    r_0^2 &= a^2 + b^2 \\
    r_1^2 &= a^2 + 4b^2 \\
    \sin \theta_0 &= \frac{b}{r_0} \\
    \cos \theta_0 &= \frac{a}{r_0} \\
    \sin \theta_1 &= \frac{2b}{r_1} \\
    \cos \theta_1 &= \frac{a}{r_1}
\end{align*}
\]

We assume that the surface on which the block is oscillating moves in accordance with the law:

\[x = \alpha \sin(\omega t),\]

hence, forcing the block oscillations.

The kinetic and potential energies of the block and of the pendulum are as described in Eqs. (3) and (4).

\[
\begin{align*}
    T_M &= \frac{M}{2} \left[ (\dot{x} - r_0 \dot{\phi} \sin(\pm \varphi + \theta_0))^2 + (r_0 \dot{\phi} \cos(\pm \varphi + \theta_0))^2 \right] + \frac{J}{2} \dot{\phi}^2 \\
    T_m &= \frac{m}{2} \left[ (\dot{x} - r_1 \dot{\phi} \sin(\pm \varphi + \theta_1) + l \dot{\psi} \cos \psi)^2 + (r_1 \dot{\phi} \cos(\pm \varphi + \theta_1) + l \dot{\psi} \sin \psi)^2 \right]
\end{align*}
\]

Figure 3: Scheme of notations of the system rocking block – pendulum.
\[
\begin{align*}
V_M &= Mg \left( a \sin(\pm \varphi) + b \cos \varphi \right) \\
V_m &= mg \left( a \sin(\pm \varphi) + 2b \cos \varphi - l \cos \psi \right)
\end{align*}
\] (4)

After Lagrangian derivation, we finally obtain the equations of motion:

\[
\begin{align*}
(M+m) \ddot{\psi} + m \left( a \sin(\varphi - \psi) + 2b \cos (\varphi - \psi) - 2 \sin (\varphi - \psi) \right) \ddot{\varphi} + m \left(1 + m\right) \cos \varphi + \left( M + 2m \right) \cos \psi - \left( M + 2m \right) \sin \psi &= 0 \\
-2a \cos (\varphi - \psi) \ddot{\varphi} + (2b - 2 \sin (\varphi - \psi)) \ddot{\varphi} + (2b - 2 \sin (\varphi - \psi)) \ddot{\psi} - m \omega^2 \sin(\alpha) \cos \psi + g \sin \psi &= 0
\end{align*}
\] (5)

Now, equation (5) can be solved, by numerical integration. It has been done by the present authors, with the collaboration of Russian colleagues from MGU, Moscow. This work has been recently submitted to the Journal of Sound and Vibration, hence results are not reported here. In the following section we want to show instead the results of some multi-body simulations we are carrying out, on the same system. We can anticipate that up to now, numerical and simulated results are in perfect accordance.

3 RESULTS

In this section, we present the results of some numerical simulations we have carried out of rocking blocks equipped with pendulums.

From a numerical point of view, a model of a rocking block connected with a pendulum leads to a multi-body problem involving both bilateral constraints (the hinge between the pendulum and the block) and unilateral contacts that might experience impacts and stick-slip phenomena and thus, requires numerical schemes for non-smooth dynamic problems. A conventional strategy for the solution of such a class of problems is based on the regularization of discontinuous terms, which are approximated by Lipschitz-continuous mollifiers. This casts the original problem into conventional Ordinary Differential Equations (ODEs) or Differential Algebraic Equations (DAEs) that can be solved by well-known numerical integrators. However, a drawback of such regularization approaches is that regularization could lead to extremely stiff functions that hinder the efficiency of ODE/DAE solvers; consequently, very short time steps or sophisticated implicit integrators are required. Therefore, as an alternative to regularization, we use a more advanced mathematical framework that deals directly with the discontinuous nature of friction and contacts, expressing the multi-body problem with the tools of Differential Variational Inequalities (DVI), Stewart and Pang (2008). In our Chrono::Engine multi-body simulation software we endorse the DVI formulation and thus, obtain high computational efficiency, good robustness and numerical stability when simulating problems of multi-body frictional contacts. The DVI problem is solved by means of a time-stepping scheme, which requires the solution of a convex second-order Cone Complementarity Problem (CCP) at each time step. In general, CCP problems include the more popular Linear Complementarity Problems (LCPs) as sub-cases; as for LCPs there are theoretical results for the existence and uniqueness of the solution under mild assumptions, Anitescu and Tasora (2010). We solve the CCPs using either a fixed-point iteration, Tasora and Anitescu (2011) or a Spectral Projected Gradient (SPG) Barzilai-Borwein method.

Impacts between rigid shapes can be handled via the introduction of restitution coefficients, but if preferred, our software can also handle the case of non-rigid frictional contacts, which fit in the broad context of DVIs. More details on this can be found, for instance, in the work by Negrut et al. (2012).

The three-dimensional simulation of the rocking block has been performed by introducing three rigid bodies, namely: the moving floor, the block and the pendulum; this being connected by a spherical joint placed at the top of the block. In addition, two box collision shapes have been assigned to the block and to the fixed floor. A collision detection algorithm finds the contact points at each time step and feeds them into the CCP solver, for advancing the DVI integration. A friction model of Amontons-Coulomb type is associated with each
contact point; thus, automatically taking into account the stick slip effects. In the presented simulations, we used static and dynamic friction coefficients $\mu_s = \mu_d = 0.6$.

The density of the simulated blocks is 2028 kg/m$^3$, the motion of the floor is defined via a rheonomic constraint that imposes a harmonic horizontal motion along the horizontal Z axis. In order to study the system, we used a cosine wave with frequency $f_z = 2.5$ Hz and amplitude $A_z = 0.015$ m. Optionally, accelerograms can be assigned to all three directions of the floor, thereby simulating a real earthquake. A scheme of the simulation environment is illustrated in Figure 4.

Various ratios of pendulum lengths, masses and horizontal frequency have been simulated, obtaining results that, although not exhibiting a perfectly steady-state periodic pattern in all cases because of the numerical nature of the simulation, can confirm the prediction of the analytical model regarding the beneficial effect of the pendulum. The studied cases are summarized in Table 1, that reports C1,…,C5 cases in which the parameters are the ratio between pendulum and block mass (expressed in percent), and the ratio between the pendulum length and the block height. Obviously, ranges of reasonable values from the applicative point of view have been chosen for these parameters.

<table>
<thead>
<tr>
<th>Table 1: Cases studied in the multi-body simulations.</th>
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<tbody>
<tr>
<td>Pendulum on block mass ratio (%)</td>
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<tr>
<td>-----------------------------------</td>
</tr>
<tr>
<td>C1 5</td>
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<tr>
<td>C2 10</td>
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<tr>
<td>C3 20</td>
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<tr>
<td>C4 30</td>
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<td>C5 40</td>
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Figure 4: The Chrono::Engine multi-body simulation environment, 4 configurations of the system.
Figure 5 summarizes the main results, i.e. the block-table relative motion during the simulation. The height on width ratio (slenderness) of the block is in this case equal to 4.2. It is shows that for a low mass pendulum (i.e. up to 20% of the block mass) no oscillations reduction occurs despite the pendulum length; when the mass pendulum is at least 30% of the block mass, a very good reduction of the block oscillations is achieved (the black line is the motion of the block with no pendulum). It is worth to notice that the effect of length of the pendulum is negligible, being the pendulum mass the stronger parameter for this frequency of “table” oscillation. Note that due to friction sensitivity of the system, case C2 presents the overturning of the block.

![Figure 5: Block-table relative motion](image)

4 CONCLUSIONS

The so-called “rocking block” problem has been investigated under different points of view, as discussed in the introduction of this article.

Before plotting a synthesis of the obtained results, it is important to state that this is a first step into a field that is not yet well explored. In fact, while several works deal with the “rocking block” problem, none of them explores the possibility of adding a pendulum to the rocking block with the aim of controlling the oscillations.

Here we presented the results of our simulations on system with forced oscillations and friction. It is of great interest to notice that the presence of the pendulum greatly reduces the ampli-
tude of vibrations in those cases, when the frequency of excitation is not within the resonance area of pendulum and for a pendulum mass that is, in our case, at least 30% of the block mass. This means that it would be possible to study some passive tuned pendulum to be added to real “rocking blocks” like ancient towers, etc., in order to reduce oscillations induced by wind, earthquakes, and so on.

Furthermore, the multi-body dynamical simulation shows that the pendulum length does not play a significant role in the reduction of oscillations, rather it has to be tuned against some dangerous resonance of the pendulum itself.

In a future perspective, the same authors are working to “draw” a map of many possible combinations of realistic forcing frequency, masses and pendulum length.

REFERENCES